Prof. David Faiman [Version 3.1]

SOLAR THERMAL COLLECTORS

In lecture 3 we saw how the window of a building can be used as a solar energy collector for winter-heating purposes. We also saw how, more generally, all of the components of the building can transfer energy from an outer surface to an inner (or *vice versa*) in a manner that is mathematically equivalent to Ohm's law in electrical circuit theory. Because the temperature differences between the interior and exterior of a building are always relatively small, I argued that the transfer of radiation through a window can also be treated, in first approximation, as if it were proportional to the temperature difference. In lecture 4, however, we saw that radiation exchange goes as the difference of the *4th power* of the temperature. Therefore, before proceeding to the higher-temperature situation of solar water heaters, let me first show how these fourth powers can reduce to an effective first power at low temperatures.

Linear Approximation to the Radiation Contribution

Most solar collectors operate over a sufficiently restricted range of working temperatures that one may write for the radiative part of the energy exchange:

$$q_{rad} = h_{rad}\,\Delta T = h_{rad}\,(T_2 - T_1) \qquad \textbf{(5.1)}$$

where the *radiative conductance* h_{rad} is defined by:

$$h_{rad} = \varepsilon\sigma \frac{T_2^4 - T_1^4}{T_2 - T_1} = 4\,\varepsilon\sigma T^3\left[1 + \left(\frac{\Delta T}{2T}\right)^2\right] \qquad \textbf{(5.2)}$$

where $T = (T_1 + T_2)/2$ is the mean operating temperature.

Example 1: Suppose a solar collector absorber plate is at 50 °C and the cover glass temperature is 20 °C. If we take an effective emittance value of $\varepsilon = 0.84$ (which we calculated in example 6 at the end of lecture 4) then, from eq (5.2), $h_{rad} = 5.58$ W m^{-2} K^{-1}.

It is similarly easy to verify that this value of the effective radiative conductance will vary by only ± 5% if the absorber temperature is raised or lowered by ± 10 °C. And, of course, we must also add the conductive and convective contributions as discussed in lecture 2. We therefore see that the total thermal conductance of a solar collector can remain reasonably constant over a considerable range of working temperatures. For those situations in which the range is too large to allow this (e.g. for solar collectors used to generate high temperatures) we can often split the "curved" total conductance, or heat loss coefficient, into a number of smaller ranges within each of which a different but constant value can be employed.

The Combined Contributions

The important conclusion from the above brief review of heat transfer mechanisms is that, to a first approximation, the overall heat loss from a solar collector is linear with temperature. Indeed, for low temperature collectors, such as the flat plate variety used in domestic water heaters, a linear efficiency curve contains all of the information needed for effective system design.

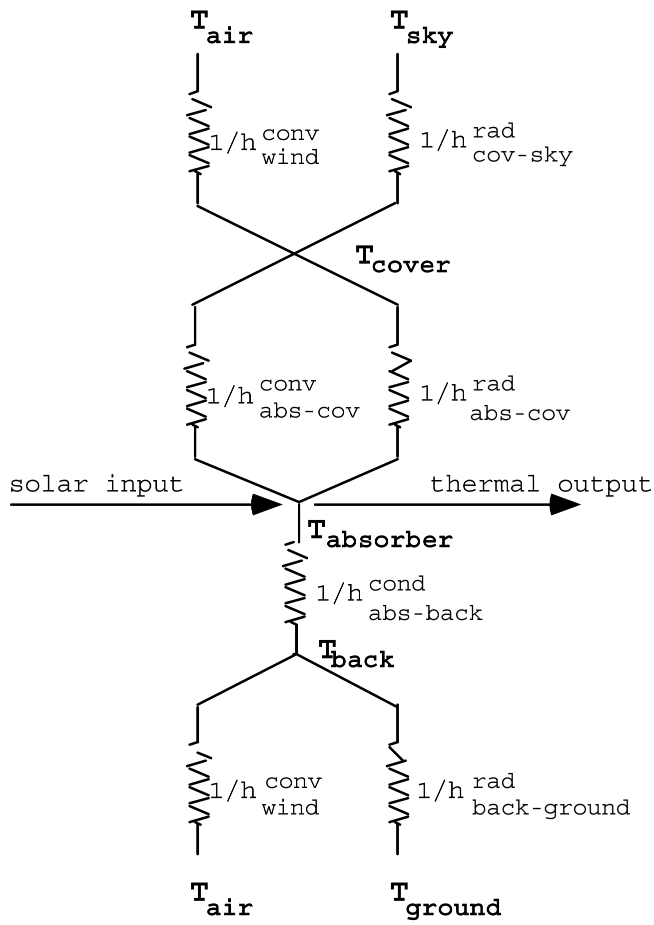

Figure1: Full thermal network representing heat balance on a flat-plate collector

For more-detailed solar collector-design purposes, thermal resistances may be combined in series and parallel in precisely the same manner that their electrical equivalents are combined. **Fig. 1** shows a network that represents the energy flows associated with a single-glazed flat-plate collector. Each of the conductances can be calculated and the resulting heat flows enable the collector to be designed so as to maximize, in principle, its efficiency for the desired end use. In practice, commercially available collectors tend not to have been designed too carefully. As a result, the kind of analysis outlined above could be used to improve their performance.

In **Fig. 1** the rear surface of the collector radiates heat to the ground and also loses heat by convection to the surrounding air. The collector back gains heat, however, by conduction from the absorber plate with which it is in contact. This absorber plate also loses heat to the cover glass by radiation and convection, and the glass loses heat to the surroundings by convection to the ambient air and by radiation to the sky. It should be noted that the effective sky temperature for radiation purposes can be substantially below the ambient air temperature - particularly in cloudless desert conditions.

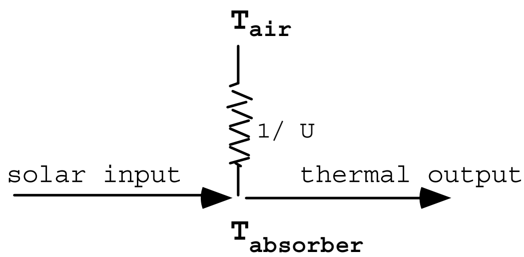

Figure 2: Simplified thermal network for heat balance on a flat-plate collector

After a full analysis of the kind represented in **Fig. 1** has been performed, the various parameters may be "lumped" together to form an overall heat transfer coefficient U for the collector. The resulting, simplified, thermal network is shown in **Fig. 2** and may be verified via an outdoor test experiment.

Collector Efficiency

Having now studied the various mechanisms by which a solar collector may exchange energy with its surroundings, both thermally and optically, we are in a position to be able to discuss its efficiency. In the literature one usually encounters the so-called *instantaneous efficiency* of a collector, which takes the functional form:

$$\eta = \eta_o - U(T_{abs} - T_{amb}) / I \qquad \textbf{(5.3)}$$

where η_o is the *optical efficiency,* (independent of temperature), U is the combined *heat loss coefficient* for the collector (i.e. combining conductive, convective and radiative losses), T_{abs} is the instantaneous absorber temperature when the ambient temperature is T_{amb} and the insolation is I. It is important to remember that the apparent linearity of eq. (**5.3**) may be deceptive since U will in general be a function of the absorber temperature. However, as emphasized above, U will often be effectively constant over the restricted range of temperatures at which a given solar collector operates.

As it stands, eq. (**5.3**) is not very useful, as it is difficult to measure the temperature of the absorber itself. One usually knows only the temperature of the fluid with which it comes in contact. We therefore, define a *collector efficiency factor* F_m as:

$$F_m = R_{abs,amb} / R_{fluid,amb} \qquad \textbf{(5.4)}$$

where $R_{abs,amb}$ is the thermal resistance (taking conduction, convection and radiation effects in parallel) between the absorber and ambient, and $R_{fluid,amb}$ is the thermal resistance between the fluid and ambient (treating the conductive resistance between fluid and absorber and $R_{abs,amb}$ as series resistances). It is easy to prove that the efficiency of the collector can now be re-written in the form:

$$\eta = F_m [\eta_o - U(T_m - T_{amb})] / I \qquad \textbf{(5.5)}$$

where T_m is the mean temperature of the *fluid* in the collector.

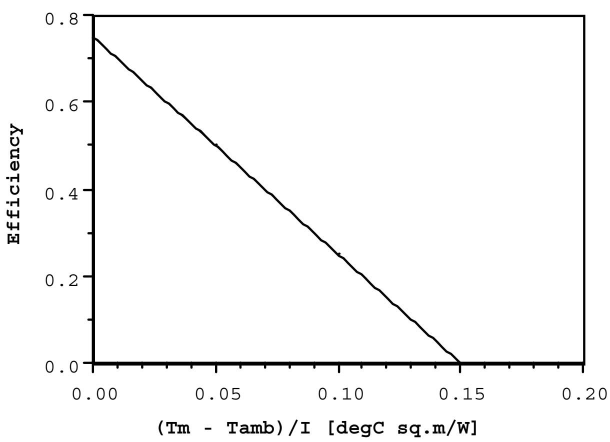

Figure 3: Instantaneous efficiency graph for a typical flat-plate collector

In many circumstances, T_m may be quite well approximated by the arithmetic mean of the in-going and out-coming fluid temperatures, both of which are easy to measure. In such cases, eq. (**5.5**) is a convenient description since it is relatively independent of the fluid flow rate. **Fig. 3** shows a typical efficiency plot for a flat plate collector, $-F_mU$ being the slope of the curve, which is highly linear over this range of temperatures, and $F_m\eta_o$ being its intercept on the efficiency axis.

Eq. (**5.5**) can be further manipulated into a form involving only the (easily measurable) fluid inlet temperature T_{in}, namely:

$$\eta = F_{in} [\eta_o - U(T_{in} - T_{amb})] / I \tag{5.6}$$

where the *heat transfer factor* F_{in} can be shown to equal:

$$F_{in} = \frac{\dot{m} C}{UA} \left[1 - e^{\frac{-F_m UA}{\dot{m} C}} \right] \tag{5.7}$$

where $\dot{m}$ is the mass flow rate of the fluid, C is its heat capacity (at constant pressure) and A is the collector (entrance) aperture area. However, owing to the exponential dependence of F_{in} on the flow rate, any efficiency graph expressed in the form of eq. (**5.7**) must specify the flow rate at which the measurements were performed.

In order to derive eq. (**5.7**), we consider the energy balance on a strip of collector of length Δy and width w:

$$\dot{m} C \Delta T = F_m \left[\eta_o - \frac{U}{I} [T(y) - T_{amb}] \right] I w \Delta y \tag{5.8}$$

Proceeding to the infinitesimal limit, the resulting differential equation has the solution:

$$T(y) - T_{amb} - \frac{\eta_o I}{U} = const \times \exp\left(\frac{-F_m U w}{\dot{m} C} y \right) \tag{5.9}$$

From the boundary condition $T(y) = T_{in}$ at y=0, we obtain the value of the constant of integration:

$$const = T_{in} - T_{amb} - \frac{\eta_o I}{U} \tag{5.10}$$

And hence, at y=L, where $T(y) = T_{out}$, we obtain:

$$\frac{T_{out} - T_{amb} - \eta_o I/U}{T_{in} - T_{amb} - \eta_o I/U} = \exp\left(\frac{-F_m\ U\ A}{\dot{m}\ C}\right)$$

(5.11)

where the product wL has been replaced by the collector area A. T_{out} can be eliminated from eq. (**5.11**) by using the definition of instantaneous efficiency:

$$\eta = \frac{\dot{m}\ C\ (T_{out} - T_{in})}{I\ A}$$

(5.12)

and the result can be re-arranged so as to resemble eq. (**5.6**). Eq. (**5.7**) is then seen to be the condition for this resemblance to be a full equality. Eq. (**5.6**) is sometimes referred to as the *Hottel-Whillier-Bliss equation* [**1,2**].

"No-dump" solar energy systems

As an instructive example of the use of the various equations derived above, we consider a daytime-only industrial plant that requires feed water at a constant temperature and flow rate. Suppose we were to install a set of solar collectors, as in **Fig. 4**, having a total aperture area *just* capable of heating mains water, under conditions of *maximum* solar radiation, at the flow rate required by the process, to the temperature required by the plant. This is called a "no-dump" system [**3**] because all of the energy it produces is useful energy – none need be dumped.

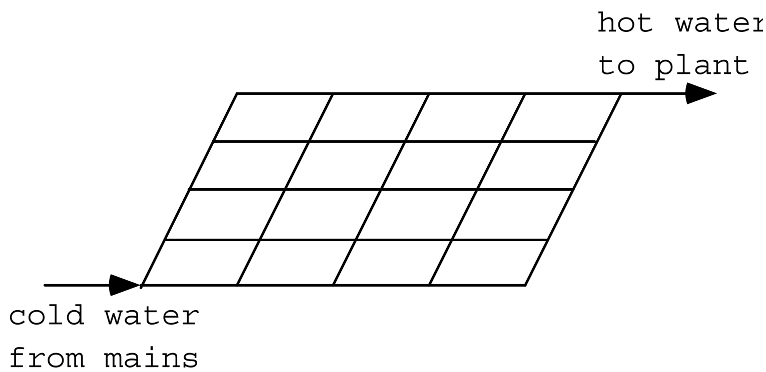

Figure 4: Cold water from the mains enters a solar collector array of area A which is to be sized so that the water exits at its maximum desired temperature only at times when the solar irradiance is at its maximum value

Let us compute what the no-dump collector area is. Suppose the plant requires water to be heated from a temperature T_{mains} to the desired process temperature T_{plant}. The required heating power is:

$$P = \rho C \dot{V} \left(T_{plant} - T_{mains} \right) \tag{5.13}$$

which, by assumption, is to be supplied by the collector array under conditions of *peak insolation only*. i.e.:

$$P = I_{peak} A \left[F_{in} \eta_o - F_{in} U \left(T_{mains} - T_{amb} \right) \right] \tag{5.14}$$

If, on average, the mains water temperature is equal to the annual mean ambient temperature (which is often more-or-less the case), then the second term in eq. (**5.14**) drops out and the peak power supplied by the collector array is:

$$P = I_{peak} A F_{in} \eta_o \tag{5.15}$$

where F_{in} must be specified at the plant flow rate. Usually, the collector specification will be given in terms of the flow-independent parameters $F_m\eta_o$ and F_mU, hence eq. (**5.7**) must be used in order to deduce the appropriate value of $F_{in}\eta_o$. Eqs. (**5.13**) and (**5.15**) then give us the value of the collector array area needed for a no-dump system:

$$A = \frac{-\rho C \dot{V}}{F_m U} \ln \left\{ 1 - \left[\frac{F_m U}{F_m \eta_o} \times \frac{\left(T_{plant} - T_{mains} \right)}{I_{peak}} \right] \right\} \tag{5.16}$$

For most "sunny" locations I_{peak} is about 1000 W m^{-2} . Suppose we have a plant that requires water at 80 ^oC, in a "sunny" location for which the mean ambient temperature is 18 ^oC - Mitzpe Ramon, for example. Suppose further that the process flow rate is 1 liter/sec. If we use a selective-surfaced flat-plate collector of a kind frequently found in domestic hot water systems, $F_m\eta_o = 0.75$ and $F_mU = 5.0$ W m^{-2} K^{-1}. Eq. (**5.16**) then indicates that the required array area is A = 446 m^2.

Let us now consider how much energy such a system could supply in one year. This is clearly:

$$Q = H_{ann} A \overline{K(\theta)} F_{in} \eta_o$$

(5.17)

where H_{ann} is the total annual *energy per unit area* incident on the collector aperture plane and K(θ), averaged here over the entire year, is a factor that enables us to incorporate the effect of non-normal incidence on the collectors. Eliminating $F_{in}\eta_o$ between eqs. (**5.17**) and (**5.15**) gives the required result:

$$Q = H_{ann} \frac{P}{I_{peak}} \overline{K(\theta)}$$

(5.18)

Interestingly, eq. (**5.18**) does *not* depend on the properties of any specific collector (apart from the annual mean incidence angle modifier, which tends to be rather similar for most flat-plate collectors [**3,4**]). At a latitude of 30°, the mean annual value of the incidence angle modifier typically amounts to about 0.9. For Mitzpe Ramon, the annual total energy incident on a fixed south-facing plane tilted at 30° to the horizontal is around H_{ann} = 8.2 GJ m^{-2}. Thus, for the system considered in the present example, the annual energy production amounts to Q = 1900 GJ, representing 47% of the total annual energy requirement of the plant. The remaining 53% would have to be supplied by a backup heating system that would ensure that the water always enters the plant at the desired temperature of 80 °C, no matter how much below maximum the solar intensity is.

The annual system efficiency is:

$$\eta_{ann} = \frac{Q}{A H_{ann}} = F_{in} \eta_o \overline{K(\theta)}$$

(5.19)

which, of course, does depend upon the type of collector employed. This efficiency is simply equal to the energy production per unit collector area divided by the total annual incident solar energy. In the case of the above flat-plate example this annual system efficiency is 52%.

High Temperature Solar Collectors

Flat-plate collectors can not give significant amounts of energy at temperatures above about 100 ^{o}C. In order to produce higher temperatures it is necessary to employ optical concentration with, or without, vacuum technology. Typical 2-D concentrators consist of a tubular receiver at the focus of a parabolic trough or, more generally, in the highest flux position of an ideal linear concentrator. Such trough-shaped collectors can be stationary if their concentration ratio is not too large, or would employ solar tracking for higher concentrations. For higher temperatures still one must, of course, employ 3-D geometry. This can be done via the use of a dish-shaped concentrator (such as *PETAL*, the very large parabolic dish reflector here at Sede Boqer) which can heat a spherical (or some other suitable geometry) receiver; or via the use of a field of heliostat mirrors which can concentrate light onto a central tower-mounted receiver (such as the system at the Weizmann Institute).

Evacuated Tube Collectors

As one example of a higher temperature collector we shall discuss the evacuated tube variety. These are basically of two types. Either the metal receiver is surrounded by a glass sleeve with the space between being evacuated. This kind of design requires a glass-to-metal seal at both ends so that the vacuum will remain intact as the metal receiver heats and expands. Difficult as this technological problem is to overcome, it was solved and this kind of tube design was the basis of the successful Luz systems.

The second alternative is to produce a vacuum bottle, similar in geometry to a Dewar flask but having the vacuum side of the inner surface coated with an absorbing surface, as in **Fig. 5**.

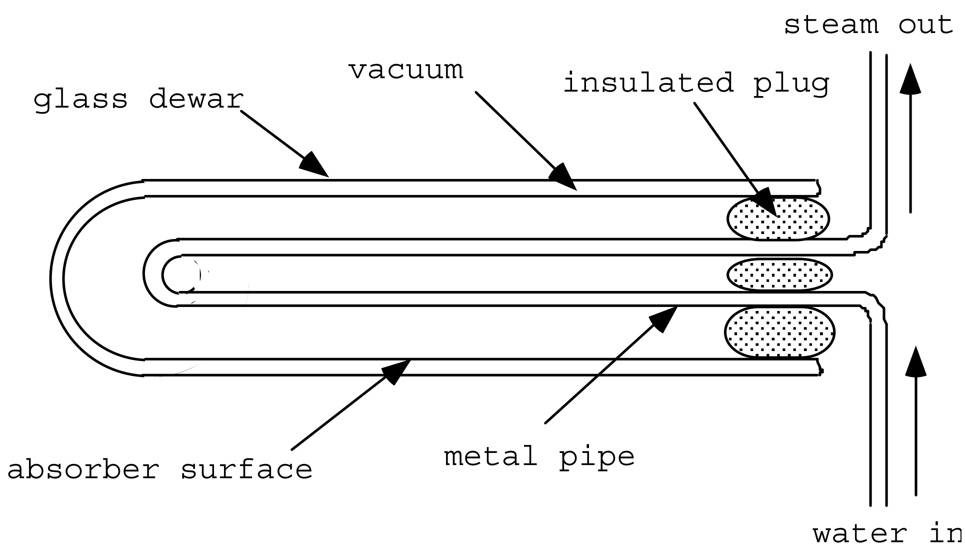

Figure 5: Schematic diagram of a Dewar-type evacuated tube solar collector

An added advantage of the Dewar-type evacuated tube, in addition to its lack of glass-to-metal seals, is the fact that broken tubes may be readily replaced without the need to dismantle the fluid heat transport system. Water is fed into one end of the tube via a metal U-pipe and exits, heated, from the same end of the collector tube: The entire collector being simply "slipped on" to the metal piping in much the same way as a shoe is placed on a foot.

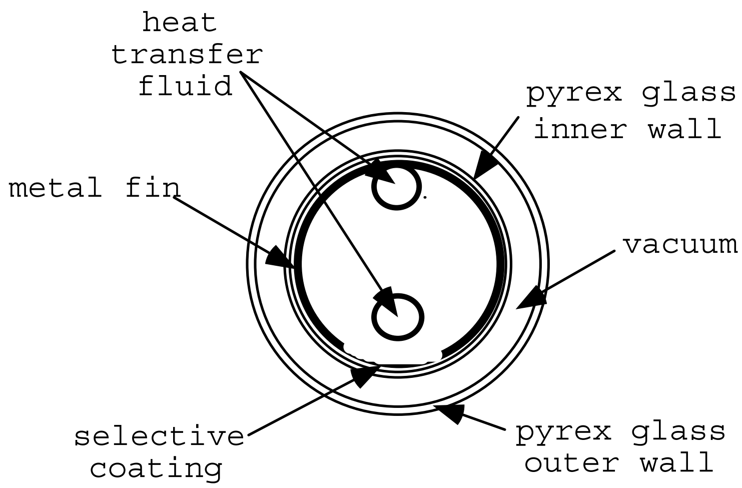

Figure 6: Cross-section through the University of Sydney evacuated tube collector

Fig. 6 shows a schematic cross section of one such collector developed at the University of Sydney in Australia [**5**]. From the figure it is clear that the absorber surface of this collector is the inner glass wall of the tube itself. In order to transfer this heat to the fluid contained in the metal piping a cylindrical metal heat fin is employed to receive energy both by radiation and, to a certain extent (since it touches the latter at only a small number of points), by direct conduction from the inner glass surface.

Let us estimate the efficiency of such a collector when exposed to the sun without any additional optical concentration. Under steady state conditions the (radiative) heat flow from the absorber surface to the outer cover glass must balance the (convective) heat loss from the latter to ambient. We have therefore:

$$\dot{Q}_{abs-cov} = A_{abs}\,\varepsilon_{eff}\,\sigma\,(T^4_{abs} - T^4_{cov}) \qquad \textbf{(5.20)}$$

which must equal:

$$\dot{Q}_{cov\text{-}amb} = A_{cov}\, h_{amb}\, (T_{cov} - T_{amb})$$

(5.21)

Rather than solve eqs. (**5.20**) and (**5.21**), iteratively, for various combinations of T_{abs} and T_{amb}, it is easier to specify various values for the heat loss and, in this manner, build up the efficiency curve. For this purpose we use the reported values [**5**] of ε_{glass} = 0.85 and ε_{abs} = 0.05, and insert them in eq. (**4.29**) of lecture 4, in order to arrive at the value ε_{eff} = 0.05. Furthermore, the evacuated tube in question has a cover-glass outer diameter of 3.8 cm and its absorber surface is of diameter 3.0 cm. For the convective heat transfer coefficient we adopt the value, suggested for this kind of situation, by Rabl [**3**] of h_{amb} = 25 W m^{-2} K^{-1}.

Let us further consider ambient conditions to consist of T_{amb} = 20 ^oC [293K] and an insolation of I = 1000 W m^{-2}, and start by considering a heat loss rate of 10 W m^{-2} from the surface of the collector tube. Inserting

$$\dot{q}_{loss} = \frac{\dot{Q}_{abs\text{-}cov}}{A_{abs}} = 10\ W\ m^{-2}$$

(5.22)

into eq. (**5.21**) yields a cover glass temperature of:

$$T_{cov} = T_{amb} + \frac{\dot{q}_{loss}\, A_{abs}}{h_{amb}\, A_{cov}} = 20.3^{\circ}\, C$$

(5.23)

Eq. (**5.20**) may now be used to compute the absorber temperature:

$$T_{abs} = \left(T^4_{cov} + \frac{\dot{q}_{loss}}{\sigma\, \varepsilon_{eff}} \right)^{1/4} = 50.3^{\circ}\, C$$

(5.24)

Under these conditions the collector heat loss coefficient relative to the *absorber surface area* is:

$$U_{abs} = \frac{\dot{q}_{loss}}{(T_{abs} - T_{amb})} = 0.33\ W\,m^{-2}K^{-1}$$

(5.25)

These calculations may be repeated for other specified values of the heat loss up to a maximum value equal to that of the incoming solar radiation, at which point the collector will have reached its so-called *stagnation temperature*. For I = 1000 W m^{-2} , the maximum collector heat loss per unit absorber surface area is about 318 W m^{-2}. **Table 1** lists the results of such calculations for various heat loss rates up to the vicinity of this value.

$\dot{q}_{loss}$ [W/sq m]	T_{cov} [degC]	T_{abs} [degC]	U_{abs} [W/m^2/K]
10	20.3	50.3	0.330
50	21.6	125.3	0.475
100	23.2	182.3	0.616
150	24.7	223.5	0.737
200	26.3	256.4	0.846
250	27.9	284.2	0.946
300	29.5	308.3	1.041

Table 1: Typical solutions to eqs. (5.20) and (5.21) for the Sydney University evacuated tube collector

It is, however, conventional practice to refer collector parameters to the *aperture area* A, which in the case of a cylindrical tube is simply the length times the diameter. Hence, for the above worked example U = π U_{abs} = 1.04 W m^{-2} K^{-1}. The total energy balance for the collector is consequently:

$$\dot{Q} = A\,\eta_o I - A_{abs} U_{abs} (T_{abs} - T_{amb})$$

(5.26)

which, since $A_{abs}/A = \pi$, takes the now familiar form:

$$\dot{Q} = A\,\eta_o I - A\,U\,(T_{abs} - T_{amb})$$

(5.27)

where η_o is the optical efficiency. For a reported value [**5**] of α_{abs} = 0.93 and ε_{glass} = 0.92 the maximum optical efficiency to be expected, approximately equal to $\varepsilon_{glass}\,\alpha_{abs}$, is η_o = 0.86. This value corresponds to an incoming radial ray. The integrated value of η_o, over parallel rays incident on one side of the entire cylindrical absorber tube will be considerably less than this value. In practice some kind of back reflector would be used in order to return some of the missing rays. For the case of a plane white diffuse reflector [**6**] the optical efficiency turns out to be about $\eta_o = 0.65$. Dividing eq. (**5.27**) through by the aperture area A, and using the results in **Table 1** together with an optical efficiency of $\eta_o = 0.65$, we may construct the efficiency curve shown in **Fig. 7**.

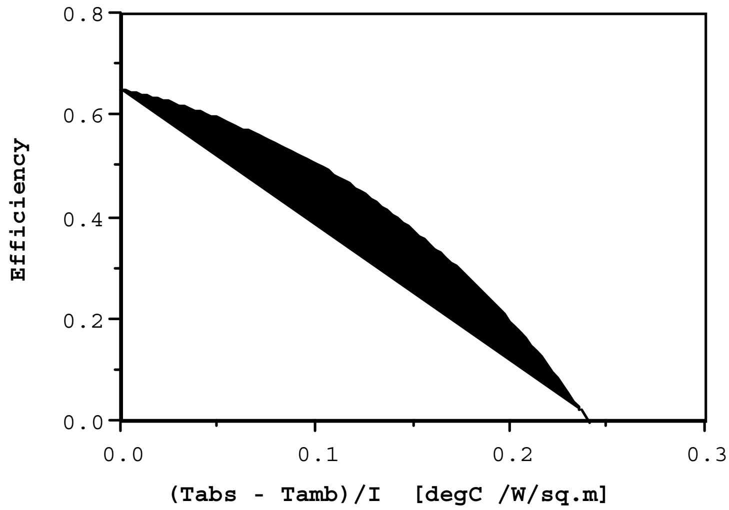

Figure 7: Modeled efficiency graph for the Univ. of Sydney evacuated tube collector

From **Fig. 7** one sees that this collector - even without optical concentration - can operate at higher temperatures than a flat-plate collector. For the flat plate collector of **Fig.3**, having $F_mU = -5.0$ W m^{-2} K^{-1} and $F_m\eta_o = 0.75$, the efficiency would fall off to zero at an abscissa of 0.15 on the graph in **Fig. 7**, whereas the evacuated tube collector still has an efficiency of about 40% at these temperatures.

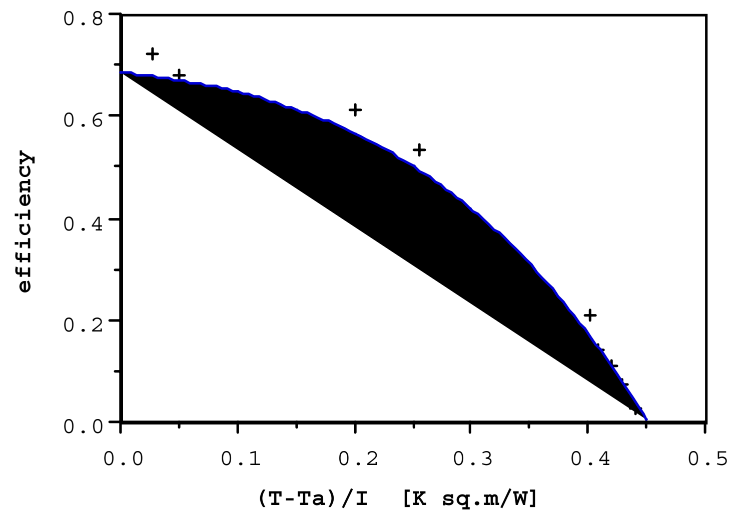

Figure 8: Experimental efficiency curve [**7**] obtained for the University of Sydney evacuated tube collector in an R=4.45 parabolic trough

Such a tubular collector can of course be operated at still higher temperatures if some form of optical concentration is employed. For example, by placing the tube at the focus of a parabolic trough reflector with concentration ratio C = 4.45 the experimental efficiency curve shown in **Fig. 8** was obtained [**7**]. From **Fig. 8** one sees that at an abscissa of 0.24 K m^{-2} W^{-1}, where the unassisted tube was seen to stagnate (i.e. reach its maximum temperature and zero efficiency), the efficiency of the tube-in-trough combination is around 50%. Indeed this system was observed to reach stagnation in the vicinity of T_{abs} = 465 ^{o}C.

APPENDIX

The Convective Heat Transfer Contribution

Convective heat transfer is by far the most difficult process to quantify, as reference must be made to many empirical correlations. One defines a *convective heat transfer coefficient* h_{conv}:

$$q_{conv} = h_{conv} (T_2 - T_1) \qquad \textbf{(5.28)}$$

with

$$h_{conv} = (\kappa/L)\, Nu \qquad \textbf{(5.29)}$$

as a definition of the dimensionless *Nusselt Number* Nu. The Nusselt number characterizes the geometry of the surface, its relationship to other surfaces and the nature of the fluid flow involved (i.e. whether *laminar* or *turbulent*). A detailed discussion of convective heat transfer coefficients is beyond the scope of this course but may be found in many books (e.g. [**3**]). Typical values of h_{conv} range from a few W m^{-2} K^{-1} for free convection in air, to more than 100,000 W m^{-2} K^{-1} in the case of condensing steam.

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[**5**] **R. Schmid, R.E. Collins** and **B.A. Pailthorpe**, Solar Energy **45** (1990) 291.

[**6**] **S.P. Chow**, **G.L. Harding**, **B. Window** and **K.J. Cathro**, Solar Energy **32** (1984) 251.

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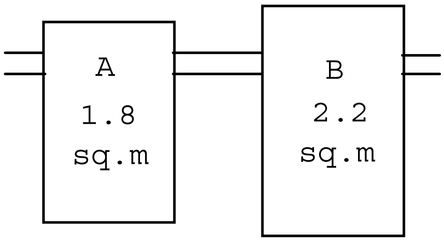
HOMEWORK PROBLEMS

1. Two collectors, A and B, are connected in series and exposed to a solar flux of 980 W m^{-2}. The ambient temperature is 26 ^oC. Collector A has an aperture area of 1.8 m^2 whereas collector B has an aperture area of 2.2 m^2.



Water is first pumped through the system at a constant flow rate of 90 litre per hour, such that it is preheated by collector A before flowing into collector B [i.e. from left to right in the diagram]. Collector A heats the water from 26 ^oC to 39 ^oC, and collector B raises the water temperature to 52 ^oC.

The flow direction is then reversed so that collector B now preheats the feed water for collector A [i.e. from right to left in the diagram]. Water entering collector B at 26 ^oC is heated to 41 ^oC and collector A then raises the water temperature to 52 ^oC.

(i) Which collector is the more efficient in each of the two flow situations?
(ii) Derive the efficiency curve (assumed linear) for each collector and sketch both graphs on the same scale.

2. An empirical form of the incidence angle modifier for a typical flat-plate collector is given by [**4**]: $K(\theta) = \cos(\theta)\,[1 + \sin^3(\theta)]$,

where θ is the angle of incidence between an incoming solar ray and the normal to the plane of the collector surface.

(i) Plot this expression as a function of θ and note its general shape.
(ii) Use the direct beam data on your Sede Boqer TMY data disk in order to calculate energy-weighted monthly average values of K(θ) for the following situations of practical interest:

(iia) A vertical, south-facing window.
(iib) A south-facing flat-plate solar collector tilted at 30^o to the horizontal.
(iic) A south-facing glazed photovoltaic panel tilted at 60^o to the horizontal.
(iid) A horizontal transparent surface (e.g. a solar pond).